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MTI Noise Integration Loss

G. V. TRUNK

Radar Analysis Staff
Radar Division

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MTI signal processing correlates the independent input noise and thus degrades detection performance when the MTI pulses are integrated. By use of simulation techniques the noise integration loss is calculated for small and intermediate sample sizes, for which the output is not Gaussian distributed. The losses for two-, three-, four-, and five-pulse MTIs are approximately 1.0, 1.8, 2.2, and 2.5 dB respectively.

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MTI NOISE INTEGRATION LOSS

INTRODUCTION

MTI signal processing correlates the receiver noise and thus results in degraded detection performance when the MTI pulses are integrated. Previous investigators [1,2] have described the decreased performance in terms of a reduction in the effective number of independent pulses integrated. However, since the effective number of pulses N_e can be represented by

$$N_e = \frac{(\sigma^2/m^2)_{\text{in}}}{(\sigma^2/m^2)_{\text{out}}},$$

where σ and m are the standard deviation and mean of the input samples, N_e has a precise meaning (in terms of detection performance) only if the output noise distribution is completely specified by N_e. For instance, when the number of pulses integrated (N) is large, the integrated output is approximately Gaussian distributed and integration improvement varies as the square root of the number of pulses integrated. Thus the loss (due to the MTI correlating the receiver noise) in signal-to-noise ratio (S/N) for a large number of integrated pulses is

$$L = 10 \log (N/N_e)^{1/2}.$$

In this report the MTI integration loss is calculated when the number of integrated pulses is small and thus the output is not Gaussian distributed. This calculation is performed using simulation techniques. First, the appropriate thresholds for a given probability of false alarm P_{fa} are calculated using importance-sampling techniques. Next, probability of detection P_D curves are generated by simulation of the pulse-by-pulse video. Finally, the MTI integration loss is found by comparing the generated P_D curves with those for independent samples [3].

FALSE-ALARM THRESHOLDS

Although Monte-Carlo simulations have been used for many years to calculate P_D curves, they have not been used to calculate P_{fa} curves because of the enormous number of repetitions usually required: approximately $10/P_{fa}$. However this difficulty can be overcome by using importance sampling [4]. The fundamental principle of the importance-sampling technique is to modify the probabilities that govern the outcome of the basic experiment of the simulation in such a way that the event of interest (the false alarm) occurs more frequently. This distortion is then compensated for by weighting each event by the ratio of the probability that this specific event would have occurred if the true probabilities had been used in the simulation to the probability that this same event would occur with the distorted probabilities. Consequently by proper choice of the distorted

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probabilities the number of repetitions can be reduced greatly. For instance, the mean of a function $Q(x)$ is given by

$$E\{Q(x)\} = \int Q(x)\, dP(x),$$

where $P(x)$ is the distribution of x. The mean of $Q(x)$ can be estimated by selecting M independent samples x_i from $P(x)$ and associating the probability $1/M$ with each event. Then $E\{Q(x)\}$ can be estimated by

$$\frac{1}{M} \sum_{i=1}^{M} Q(x_i). \tag{1}$$

The importance-sampling technique uses the Radon-Nikodyn derivative to express the mean value of $Q(x)$ by

$$E\{Q(x)\} = \int Q(x) \frac{dP(x)}{dG(x)} dG(x),$$

where $G(x)$ is a distribution function. The mean $E\{Q(x)\}$ can be estimated by selecting M independent samples from $G(x)$ and associating the probability $dP(x_i)/MdG(x_i)$ with each event $Q(x_i)$. Thus $E\{Q(x)\}$ is estimated by

$$\frac{1}{M} \sum_{i=1}^{M} Q(x_i) \frac{dP(x_i)}{dG(x_i)}. \tag{2}$$

Since (1) and (2) are both unbiased estimates of $Q(x)$, it is possible to select $G(x)$ so that the variance of (2) is less than the variance of (1).

In our problem of determining the threshold for a given P_{fa}, when MTI samples are noncoherently integrated, it is necessary to estimate the distribution curve

$$P(Z_j \leqslant T) = 1 - P_{fa}, \tag{3}$$

where

$$Z_j = \sum_{i=1}^{N} Z_{ij}, \tag{4}$$

in which

$$Z_{ij} = \left[(x_{ij}'^2 + y_{ij}'^2)/P(k)\right]^{1/2} \tag{5}$$

where, for a two-pulse MTI,

$$x'_{ij} = x_{ij} - x_{i-1,j} \tag{6}$$

and

$$y'_{ij} = y_{ij} - y_{i-1,j}, \tag{7}$$

with x_{ij} and y_{ij} being independent Gaussian variables with zero mean and a variance of σ and $P(k)$ being the noise power out of a k-pulse MTI: $P(2) = 2$, $P(3) = 6$, $P(4) = 20$, and $P(5) = 70$. The straightforward way of estimating (7) is to generate Gaussian samples by

$$x_{ij} = \sigma(-2 \ln u_{ij})^{1/2} \sin 2\pi v_{ij} \tag{8}$$

and

$$y_{ij} = \sigma(-2 \ln u_{ij})^{1/2} \cos 2\pi v_{ij}, \tag{9}$$

with u_{ij} and v_{ij} being independent random numbers uniformly distributed on the interval (0,1). To estimate (3), M independent sums $\{Z_j, j = 1, M\}$ are formed using (4) through (7), and the estimated distribution is

$$\hat{P}(Z \geqslant T) = \frac{1}{M} \sum_{j=1}^{M} \delta_j,$$

where

$$\delta_j = 1, \quad Z_j \geqslant T,$$
$$= 0, \quad Z_j < 0.$$

Importance sampling differs from the previous procedure by generating samples using

$$x_{ij} = \alpha(-2 \ln u_{ij})^{1/2} \sin 2\pi v_{ij} \tag{10}$$

and

$$y_{ij} = \alpha(-2 \ln u_{ij})^{1/2} \cos 2\pi v_{ij}, \tag{11}$$

where $\alpha > \sigma$, a device which yields more false alarms. Using (10) and (11) and (4) through (7), M sums Z_j are generated. Then the estimated distribution is

$$\hat{P}(Z \geqslant T) = \frac{1}{M} \sum_{j=1}^{M} \delta_j P_j,$$

where

$$\delta_j = 1, \quad Z_j \geqslant T,$$
$$= 0, \quad Z_j < 0,$$

and

$$P_j = \prod_{i=2-k}^{N} \frac{\frac{1}{2\pi\sigma^2} \; e^{-(x_{ij}^2 + y_{ij}^2)/2\sigma^2}}{\frac{1}{2\pi\alpha^2} \; e^{-(x_{ij}^2 + y_{ij}^2)/2\alpha^2}}.$$

With use of $\alpha = 2.0$ and $M = 20{,}000$ for $N = 4$, $\alpha = 1.7$ and $M = 10{,}000$ for $N = 8$, $\alpha = 1.5$ and $M = 10{,}000$ for $N = 16$, and $\alpha = 1.3$ and $M = 2500$ for $N = 32$, threshold curves were generated for two-, three-, four-, and five-pulse (binary weighting) MTIs and are shown in Fig. 1. The reference curve for independent samples was generated using detection curves in Robertson [3].

PROBABILITY OF DETECTION

Since the S/N out of the MTI is a function of the target doppler, the doppler frequency where the input and output S/N are equal will be used. The S/N gain (or loss) provided by the k-pulse MTI is

$$\frac{\left(\sum_{i=1}^{k} a_i \cos i\phi_k\right)^2 + \left(\sum_{i=1}^{k} a_i \sin i\phi_k\right)^2}{\sum_{i=1}^{k} a_i^2}, \tag{12}$$

where $\{a_i, i = 1, \ldots, k\}$ are the MTI coefficients and ϕ is the change in target phase between successive PRFs. Setting (12) equal to 1 and solving for ϕ_k yields the solutions $\phi_2 = 90°$, $\phi_3 = 103°$, $\phi_4 = 110.9°$, and $\phi_5 = 116.5°$.

Thus the P_D for a k-pulse MTI and a given P_{fa} can be found by generating sample video using

$$x_{ij} = \sigma(-2 \ 1n \ u_{ij})^{1/2} \sin 2\pi v_{ij} + A \sin i\phi_k \tag{13}$$

and

$$y_{ij} = \sigma(-2 \ 1n \ u_{ij})^{1/2} \cos 2\pi v_{ij} + A \cos i\phi_k, \tag{14}$$

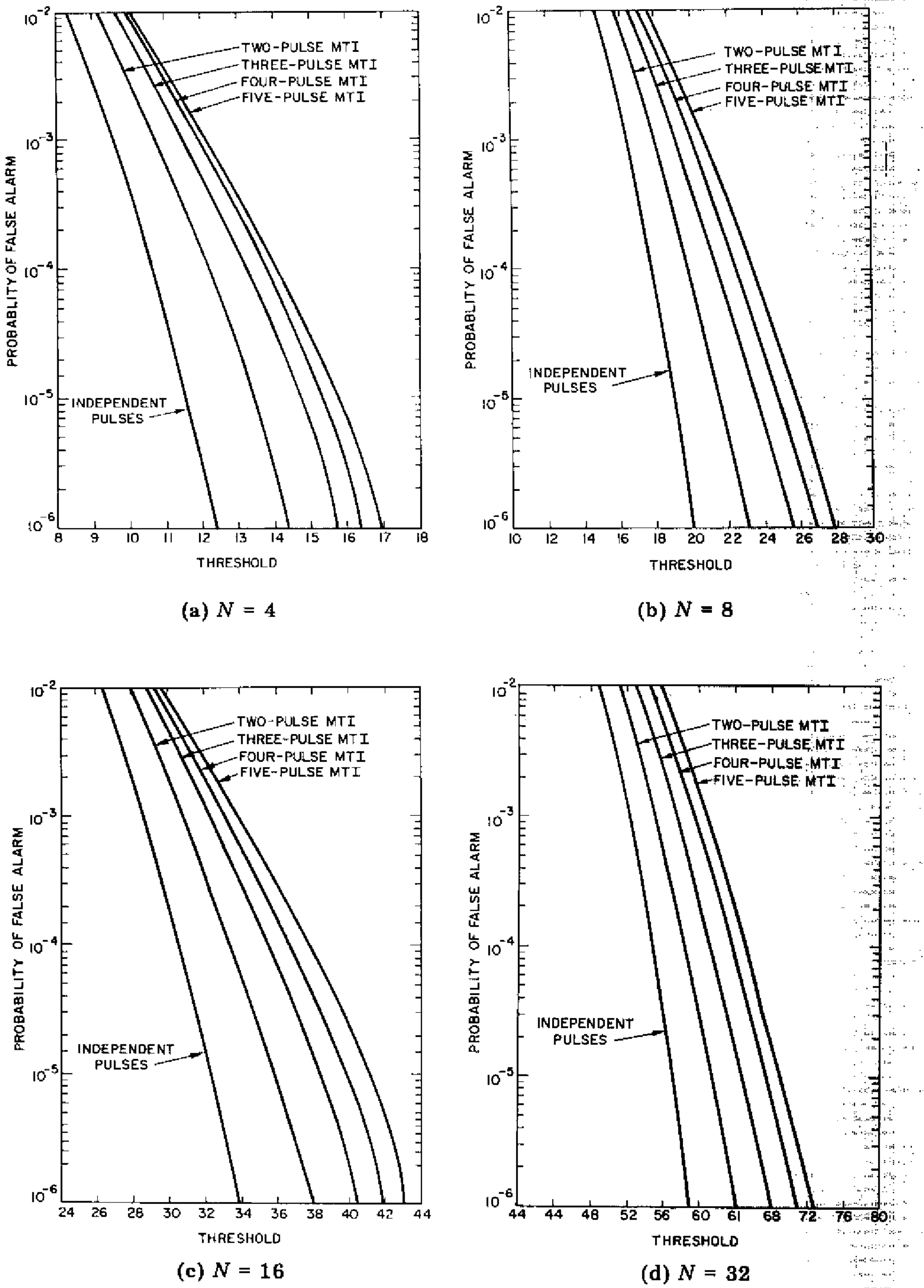
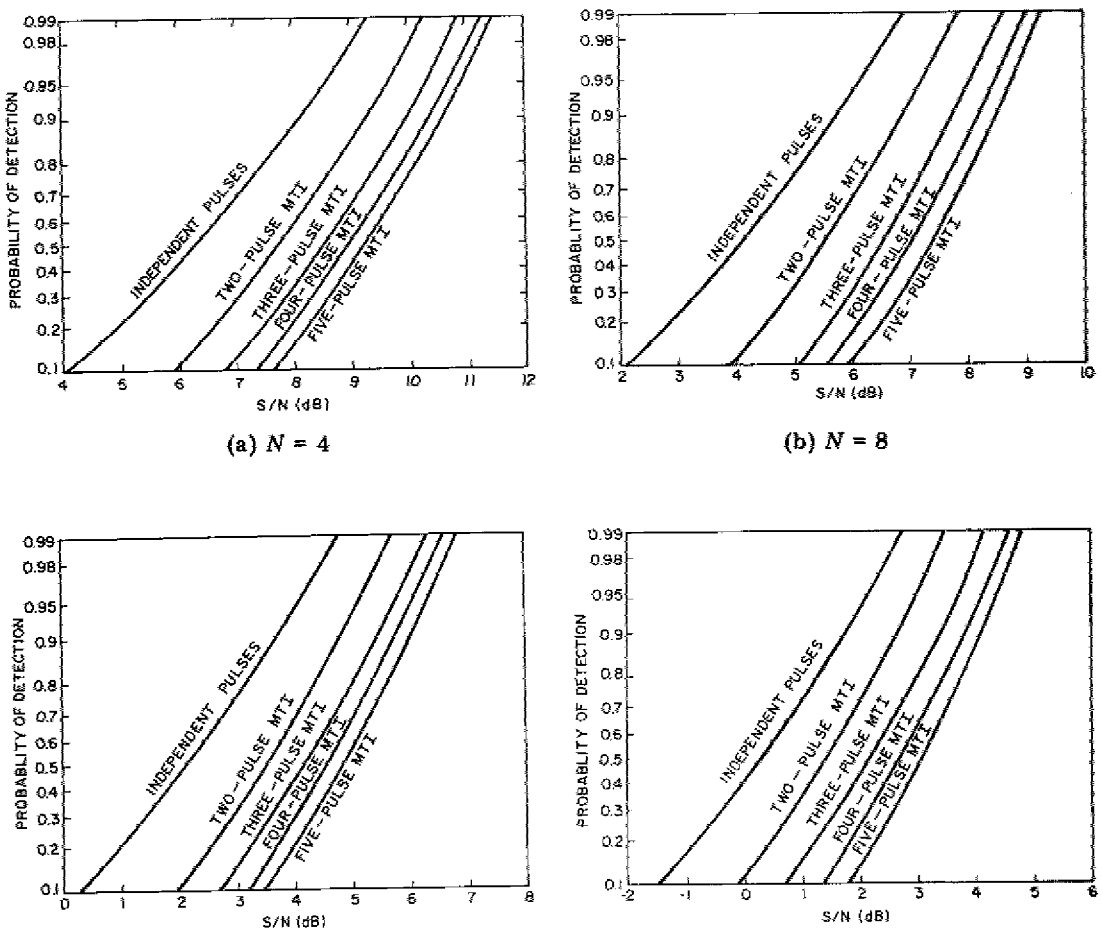


Fig. 1 — Threshold curves for N pulses integrated

where S/N(dB) = 10 log $(A^2/2\sigma^2)$. By use of (13) and (14) and (3) through (7), M = 1024 Z_j values were generated for each S/N and compared to the appropriate threshold. The P_D curves for $P_{fa} = 10^{-6}$ are shown in Fig. 2.

The difference between the P_D curves for the various MTIs and the curve for independent pulses is the MTI noise integration loss. This loss is given in Table 1 for the P_D and P_{fa} values indicated. The loss appears to be fairly independent of both N, the number of pulses integrated, and P_{fa}.



(a) $N = 4$ (b) $N = 8$

(c) $N = 16$ (d) $N = 32$

Fig. 2 — Probability of detection curves for N pulses integrated with $P_{fa} = 10^{-6}$

Table 1 — MTI Noise Integration Loss for $P_D = 0.9$ and N Noncoherent Pulses Integrated

MTI Pulses	Loss (dB)				Average Difference (dB)
	$N = 4$	$N = 8$	$N = 16$	$N = 32$	
$P_{fa} = 10^{-6}$					
Two	1.1	1.1	1.1	0.9	1.0
Three	1.8	1.9	1.7	1.7	1.8
Four	2.2	2.4	2.1	2.1	2.2
Five	2.5	2.7	2.3	2.4	2.5
$P_{fa} = 10^{-4}$					
Two	1.1	0.9	0.9	0.8	0.9
Three	1.8	1.7	1.6	1.5	1.6
Four	2.1	2.1	1.9	1.9	2.0
Five	2.3	2.5	2.1	2.2	2.3

COMPARISON WITH PREVIOUS RESULTS

The number of effective pulses integrated for a k-pulse MTI is given [1] by

$$N_e(k) = \frac{N^2}{N + 2 \sum_{J=1}^{N-1} (N - j) R_k^2(j)},$$

where $R_k(j)$ is the correlation coefficient

$$R_k(j) = \frac{E\{x_i' \, x_{i+j}'\}}{P(k)}.$$

Thus, to find the MTI noise integration loss, the difference must be found between the required S/N for N_e and N independent pulses. To accomplish this, a curve of S/N versus N for $P_D = 0.9$ and $P_{fa} = 10^{-6}$ was generated using the detection curves in Robertson [3] and is shown in Fig. 3. From this curve the MTI noise integration loss was calculated and is shown in Table 2. These losses are about 0.2 dB higher than the corresponding losses in Table 1.

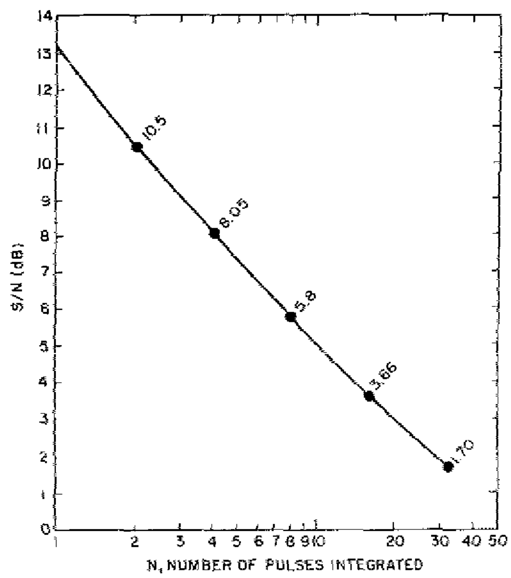


Fig. 3 — S/N for $P_D = 0.9$ and $P_{fa} = 10^{-6}$ as a function of the number of independent pulses integrated

Table 2 — MTI Noise Integration Loss Using the Effective Number of Pulses N_e Integrated for $P_D = 0.9$ and $P_{fa} = 10^{-6}$

MTI Pulses	Loss (dB)				Average Difference (dB)
	$N = 4$	$N = 8$	$N = 16$	$N = 32$	
Two	1.1	1.2	1.2	1.1	1.1
Three	1.8	1.7	1.9	1.8	1.8
Four	2.4	2.6	2.4	2.3	2.4
Five	2.7	2.9	2.9	2.7	2.8

SUMMARY

MTI signal processing correlates the receiver noise, and this results in an MTI noise integration loss. The losses for two-, three-, four-, and five-pulse MTIs are approximately 1.0, 1.8, 2.2, and 2.5 dB respectively. The P_D for a given target can be found using the following procedure:

1. Calculate the input S/N (to the MTI) using the radar range equation;
2. Calculate the output S/N from the MTI using (12)
3. Use Fig. 2 to determine P_D or else assume all N pulses are independent, reduce S/N by the MTI noise integration loss, and find P_D from standard detection curves such as given in Robertson [3].

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